WELMEC 6.9

Issue 1

# WELMEC

**European cooperation in legal metrology**

# PREPACKAGES - UNCERTAINTY OF MEASUREMENT

## Guidelines to estimate the uncertainty of measurement when determining the actual quantity of product in prepackages

June 2009

# WELMEC

**European cooperation in legal metrology**

WELMEC is a cooperation between the legal metrology services of the Member States of the European Union and EFTA.

This document is one of a number of Guides published by WELMEC. The Guides are purely advisory and do not themselves impose any restrictions or additional technical requirements beyond those contained in relevant EC Directives.

Alternative approaches may be acceptable, but the guidance provided in this document represents the considered view of WELMEC as to the best practice to be followed.

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## General

Member states of the European Economic Area have implemented the Council Directives 75/106/EEC of 19 December 1974 and 76/211/EEC of 20 January 1976 in their national legislation. These directives deal with marking and quantity control of e-marked prepackages.

These directives have recently been amended by Directive 2007/45/EC of 5 September 2007, which revokes Directives 75/106/EEC and 80/232/EEC and amends Directive 76/211/EEC. The only prescribed quantities permitted without specified time limit relate to certain wines and spirits. Members States can retain prescribed quantities for milk, butter, dried pasta and coffee until 11 October 2012, and for sugar until 11 October 2013.

This document is part of a series of documents (to be) published by WELMEC:

6.0 Introduction to WELMEC documents on ‘e’ marked prepackages
6.1 Definitions of terms
6.2 Translations of terms
6.3 Guidance for the Harmonised Implementation of Council Directive 76/211/EEC as amended
6.4 Guide for packers and importers of ‘e’ marked prepacked products
6.5 Guidance on Controls by Competent Departments
6.6 Guide for recognition of procedures
6.7 Guide for Market Controls on Prepackages for Competent Departments
6.8 Guidance for the Verification of Drained Weight
**6.9 Prepackages - Uncertainty of Measurement**
6.10 Controls on non-e-marked Prepackages

Some of these documents represent the opinion of WELMEC, others are under revision or preparation. Those that have been agreed by WELMEC are published on their website (www.welmec.org).

This series of documents primarily intends to provide guidance to all those concerned with the application of Directive 2007/45/EC laying down the rules on nominal quantities for prepacked products, repealing Council Directives 75/106/EEC and 80/232/EEC, and amending Council Directive 76/211/EEC (the Directives). They are intended to lead to a uniform interpretation and enforcement of these directives and assist in the removal of barriers to trade

## Disclaimer

Please note that this series of documents does not deal with all the matters not covered by the above directives, such as requirements for certain products to be made up in prescribed quantities, and controls on non e-marked prepackages.

# 1. **INTRODUCTION**

This document presents a simplified guideline with examples for evaluating and expressing uncertainty in measurement in the area of prepackages. Primarily it should serve competent departments, which perform checks of prepacked products in accordance with the Directive 76/211/EEC.

The document refers to the international document on uncertainty of measurement (GUM: Guide to expression of Uncertainty in Measurement from ISO dated 1993 and EA 4/02).

Uncertainty of measurement is not mentioned in the directive 76/211/EEC, instead the term *error* is used. In order to comply with Annex II.1, paragraph 2 of the directive, competent departments must ensure that "**the error made in measurement of the actual contents of prepackages shall not exceed one-fifth of the tolerable negative error for the nominal quantity in the prepackages**. " This requirement does not apply to packers.

However the information provided in this guide could be useful for packers who want to have their procedures (internal system of quantity control of prepackages) recognized.

Competent departments that are accredited are subject to requirements in relation to measurement uncertainty. Whenever this guide is at variance with these requirements, accreditation requirements prevail.

Definitions in the International Vocabulary of Basic and General Terms in Metrology determines that:

- **Uncertainty of Measurement** is a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand;

- **Error** is result of measurement minus a true value of the measurand.

In metrology it is usual to give the result of a measurement in the notation of an average value and uncertainty followed by the unit, as in this example for mass[a]: m = 100,2 g ± 0,5 g.

The combined expanded uncertainty of the result ($U$) is often caused by different sources. For combining the components which contribute to the overall or the expanded uncertainty, the following expression is normally used:

$$U = k \cdot \sqrt{\sum_{i=1}^{n} c_i^2 u_i^2} \qquad (1)$$

k – coverage factor, is typically in the range 2 to 3
$u_c$ – combined standard uncertainty
$u_i$ – standard uncertainty component number i
$c_i$ - sensitivity coefficient component number i

Each component ($u_i$) is calculated or estimated, for example:

- as a standard deviation of a sample, calculated using the expression

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[a] this is sometimes referred to as: 'conventional mass', 'weight'

$$u_i = \sqrt{\frac{\sum \left(x_i - \bar{x}\right)^2}{n-1}} \qquad (2)$$

for a series of $n$ repeated measurements,

- estimated, for rectangular distribution using the expression

$$u_i \approx \frac{a}{\sqrt{3}} \qquad (3)$$ [b]

in this case measurements have an equal probability of occurrence between the highest and the lowest values (*d* is the difference between the highest and the lowest result, one scale division, or other).

Symbols and designations used in the calculations:

| | |
|---|---|
| $u(m_T)$ | standard uncertainty of determination of tare |
| $u(m_G)$ | standard uncertainty of determination of prepackage gross mass |
| $u(m_N)$ | standard uncertainty of determination of prepackage net mass |
| $u(\rho)$ | standard uncertainty of determination of density of product |
| $u(WI)$ | standard uncertainty from calibration certificate of weighing instrument (NAWI) |
| $u(V_{pic})$ | standard uncertainty from calibration certificate of pycnometer |
| $n$ | number of samples or measurements |
| $s$ | standard deviation |
| $mpes$ | maximum permissible error in service |
| $mpes_0$ | maximum permissible error in service directly after zero setting (according to EN 45 501 mpe at zero by verified NAWI must be within ± 0,25 e); $mpes_0$ is relevant for NAWI class III only (EURAMET-cg-18-01.Non –Automatic-Weighing-Instruments.pdf) |
| $d$ | scale division on NAWI |
| $\rho$ | (average) density of the sample at density determination |
| $\nu$ | degrees of freedom |

Indices used in expressions:

| | |
|---|---|
| $T$ | tare |
| $N$ | net |
| $G$ | gross |
| $d$ | density |

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[b] 2a is difference between specification limits (e.g. mpes or rounding limits d/2)

## 2. CALCULATION OF UNCERTAINTY OF MEASUREMENT OF ACTUAL QUANTITY OF PRODUCT IN PREPACKAGES AT A GIVEN TEMPERATURE (T = 20 °C ± 0,5 °C)

The actual quantity of product in prepackages is checked by the reference method (statistical checking of batches of prepackages). It can be measured directly by means of weighing instruments or volumetric instruments or, in the case of liquids, indirectly, by weighing the prepacked product and measuring its density.

The uncertainty of measurement made in measuring the actual quantity of product in prepackages shall not exceed one-fifth of the tolerable negative error (TNE) for the nominal quantity of product in the prepackage.

If uncertainty of measurement exceeds one-fifth of TNE, more accurate / precise measurement instruments or measurement method must be used to determine the actual quantity of product of prepackages.

As it was already mentioned, the uncertainty variation of the result is often caused by different sources.

In case of measuring the actual quantity of product in prepackages by weighing the prepacked product and measuring its density the expression for expanded uncertainty would be as follows:

$$U_{k=2}(V) = 2 \cdot \sqrt{c(m_N)^2 u(m_N)^2 + c(\rho)^2 u(\rho)^2} \qquad (4)$$

See 2.1.5 below as to how the sensitivity coefficients, $c_i$, are calculated.

In the formula coverage factor k = 2 is used which for normal distribution corresponds to a coverage probability of approximately 95%. This coverage factor is often used in calibration of measuring instrument.

Before determining the actual quantity of product in prepackages usually the tare needs to be determined.

Below is considered the calculations of expanded uncertainty for verified and calibrated NAWI.

## 2.1. Verified NAWI

### 2.1.1. Determination of tare

Tare could be dealt with in two different ways: measuring the mass of the individual tare of each prepackage or averaging the mass of several tares.

Individual tare

The tare weight of each prepackage needs to be determined if the standard deviation of the tare mass is more than one-tenth of the TNE of the nominal quantity of the prepackage, and in all other cases when average tare is not used.

Individual tare determination:

$$u(m_T)^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + \underbrace{\left(\frac{d_T}{2\sqrt{3}}\right)^2}_{\text{rounding error on tare indication}} + \underbrace{\left(\frac{d_T}{2\sqrt{3}}\right)^2}_{\text{rounding error on zero indication}} = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + 2\left(\frac{d_T}{2\sqrt{3}}\right)^2 \text{ [c]} \tag{5}$$

for NAWI class III:

$$u(m_T)^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + \left(\frac{d_T}{2\sqrt{3}}\right)^2 + \left(\frac{mpes_0}{\sqrt{3}}\right)^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + \underbrace{\left(\frac{d_T}{2\sqrt{3}}\right)^2}_{\text{rounding error on tare indication}} + \underbrace{\left(\frac{d_T}{4\sqrt{3}}\right)^2}_{\text{error on zero}} \text{ [d]} \tag{5.1}$$

Average tare

In this case the variation in mass of the packing material must be carefully considered. This method is only suitable if the standard deviation of the tare mass is less than one-tenth[e]
From a lot of packing material of size N is taken a sample of size $n_T$ and average tare ($\overline{m_T}$) with standard deviation ($s_T$) is measured.

Uncertainty of average tare is:

$$u(\overline{m_T})^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + \underbrace{\left(\frac{d_T}{2\sqrt{3}}\right)^2}_{\text{rounding error on tare indication}} + \underbrace{\left(\frac{d_T}{2\sqrt{3}}\right)^2}_{\text{rounding error on zero indication}} + \left(\frac{s_T}{\sqrt{n_T}}\right)^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + 2\left(\frac{d_T}{2\sqrt{3}}\right) + \left(\frac{s_T}{\sqrt{n_T}}\right)^2 \tag{6}$$

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[c] in practice for NAWI class II, scale in use between 0 and 5000 e: $d_T/2 \approx 1/20\ mpes_T$ and both rounding errors are neglectable

[d] the rounding error on tare indication and the error at zero are no longer neglectable in the range 0-500 e because in this range $mpes_T = e = d_T$ and $mpes_0 = 1/4\ e = 1/4\ d$

[e] OIML R87, annex B, table B1 of TNE of the nominal quantity.
In some countries one-fourth of TNE is used.

for NAWI class III:

$$u\left(\overline{m_T}\right)^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + \left(\frac{d_T}{2\sqrt{3}}\right)^2 + \left(\frac{d_T}{2\sqrt{3}}\right)^2 + \left(\frac{d_T}{4\sqrt{3}}\right)^2 + \left(\frac{s_T}{\sqrt{n_T}}\right)^2 \qquad (6.1)$$

### 2.1.2. Determination of mass of prepackage (gross mass)

From a batch of prepackages size N, a sample of size $n_G$ is taken in accordance with Directive 76/211/EEC. The average ($\overline{m_G}$) and standard deviation ($s_G$) is calculated after weighing each prepackage.

Standard uncertainty of the determination of the mass of a single prepackage (gross mass):

$$u(m_G)^2 = \left(\frac{mpes_G}{\sqrt{3}}\right)^2 + \left(\frac{d_G}{2\sqrt{3}}\right)^2 + \left(\frac{d_G}{2\sqrt{3}}\right)^2 + \left(\frac{s_T}{\sqrt{n_T}}\right)^2 = \left(\frac{mpes_G}{\sqrt{3}}\right)^2 + 2\left(\frac{d_G}{2\sqrt{3}}\right)^2 \qquad (7)$$

for NAWI class III:

$$u(m_G)^2 = \left(\frac{mpes_G}{\sqrt{3}}\right)^2 + \left(\frac{d_G}{2\sqrt{3}}\right)^2 + \left(\frac{d_G}{4\sqrt{3}}\right)^2 \qquad (7.1)$$

### 2.1.3. Calculation of mass of the product (net mass)

In some cases the net mass of prepackage is calculated as:

Mass of the product (net mass) = mass of the prepackage (gross mass) – individual tare.

In this case the uncertainty for the mass of the product in a single package (net mass) is calculated with expression as follows:

$$u(m_N)^2 = u(m_G)^2 + u(m_T)^2 \qquad (8)$$

Usually the mass of product (net mass) is calculated as:

Mass of the product (net mass) = mass of the prepackage (gross mass) – average mass of tare.

In this case the uncertainty for the mass of a single package (net mass) is calculated with expression as follows:

$$u(m_N)^2 = u(m_G)^2 + u\left(\overline{m_T}\right)^2 \qquad (9)$$

Sensitivity coefficient at net mass determination is 1.

### 2.1.4. Determination of density of liquid product

Weighing the quantity of product in the prepackages (net mass) and then determining the density is the most accurate way of determining the quantity of product in prepackages with nominal quantity expressed by unit of volume.

The density could be determined by different methods. In the following example the calculation of standard uncertainty is appropriate for a metallic pycnometer.

**Determination of density with pycnometer**

The density $(\rho)$ is usually determined three times for one product $(n_d = 3)$ and is calculated with expression as follows:

$$\rho = 0{,}99985 \cdot \frac{m_d}{V_{pic}} + 0{,}0012 \qquad (10)$$

$m_d$ conventional mass of the sample in the pycnometer

For the further calculation of standard uncertainty the average density is used.
Standard uncertainty component from determination of density[f]:

$$u(\rho)^2 = \left(u(m_d)\cdot c(m_d)\right)^2 + \left(u(V_{pic})\cdot c(V_{pic})\right)^2 + \left(\frac{s_d}{\sqrt{n_d}}\right)^2 \qquad (11)$$

In the expression $u(m_d)$ is the standard uncertainty for the weight of the pycnometer content. The empty pycnometer is tared (handle like zero, EN 45 501 4.6.3).

$$u(m_d)^2 = \left(\frac{mpes_d}{\sqrt{3}}\right)^2 + \left(\frac{d_d}{2\sqrt{3}}\right)^2 + \left(\frac{d_d}{2\sqrt{3}}\right)^2 = \left(\frac{mpes_d}{\sqrt{3}}\right)^2 + 2\left(\frac{d_d}{2\sqrt{3}}\right)^2 \qquad (12)$$

Sensitivity coefficients c($m_d$) and c($V_{pic}$) are calculation as:

$$c(m_d) = \frac{\partial\rho}{\partial m} = 0{,}99985 \cdot \frac{1}{V_{pic}} \qquad (13)$$

$$c(V_{pic}) = \frac{\partial\rho}{\partial V} = -0{,}99985 \cdot \frac{m_d}{V_{pic}^{\ 2}} \qquad (14)$$

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[f] Uncertainty regarding measuring a temperature and a density of air have also uncertainty but they is not relevant for result because density determination is performed at 20 °C ± 0,5 °C

### 2.1.5. Calculation of combined uncertainty $u_c(V)$

$$V = \frac{m_N}{\rho} \qquad (15.1)$$

Sensitivity coefficients c($m_N$) and c(ρ) are calculation as:

$$c(m_N) = \frac{\partial V}{\partial m_N} = \frac{1}{\rho} \quad (15.2) \quad \text{and} \quad c(\rho) = \frac{\partial V}{\partial \rho} = m_N . \frac{-1}{\rho^2} \qquad (15.3)$$

Combined uncertainty:

$$u_c(V)^2 = c(m_N)^2 . \, u(m_N)^2 + c(\rho)^2 . \, u(\rho)^2 \qquad (15.4)$$

$$u_c(V) = \frac{m_N}{\rho} \cdot \sqrt{\left(\frac{u(m_N)}{m_N}\right)^2 + \left(\frac{u(\rho)}{\rho}\right)^2} \qquad (15.5)$$

### 2.1.6. Calculation of expanded uncertainty

Expanded uncertainty for the quantity of product in a single prepackage is:

$$U = k \cdot u_c \, ^{g} \qquad (16)$$

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[g] When we set limits to the various contributions, we will have unlimited degrees of freedom, therefore in practice k=2 is sufficient. Theoretically you use k=2 if effective degrees of freedom ≥ 50. If it is less than 50, calculate a coverage factor from a student-t table.

## 2.2. Calibrated NAWI

The whole procedure of determination of quantity of prepackaged products is the same as that used for the verified NAWI.

### 2.2.1. Determination of tare

Individual tare

Mass of the tare of each prepackage is determined.

$$u(m_T)^2 = u(WI_T)^2 \qquad (17)$$

The uncertainty in use should be used, that should be available from the calibration certificate and be in accordance with the EA guide EURAMET-cg-18-01. Non –Automatic-Weighing-Instruments.pdf

Average tare

From a lot of packing material of size N a sample of size $n_T$ is taken and measured, after which the average tare $(\overline{m_T})$ with standard deviation $(s_T)$ are calculated.

Standard uncertainty of average tare is:

$$u(\overline{m_T})^2 = u(WI_T)^2 + \left(\frac{s_T}{\sqrt{n_T}}\right)^2 \qquad (18)$$

### 2.2.2. Determination of mass of prepackage (gross mass)

From a lot of prepackages size N, a sample of size $n_G$ is taken of prepackages. The average $(\overline{m_G})$ and standard deviation $(s_T)$ is calculated after weighing each prepackage.

Standard uncertainty component of the determination of the mass of a single prepackage (gross mass):

$$u(m_G)^2 = u(WI_G)^2 \qquad (19)$$

### 2.2.3. Calculation of mass of product in the prepackage (net mass)

The uncertainty for the mass of product in a single prepackage (net mass) (considering average tare) is calculated with expression as follows:

$$u(m_N)^2 = u(m_G)^2 + u(m_T)^2 \qquad (20)$$

### 2.2.4. Determination of density

Density of product was measured with metallic pycnometer.

$$u(\rho)^2 = \left(u(m_d)\cdot c(m_d)\right)^2 + \left(u(V_{pic})\cdot c(V_{pic})\right)^2 + \left(\frac{s_d}{\sqrt{n_d}}\right)^2 \qquad (21)$$

$$u(m_d) = u(WI_d) \qquad (22)$$

For determination of $c(m_d)$, $c(V_{pic})$, calculation of combined uncertainty and expanded uncertainty use formula 13 – 16.

## 3. EXAMPLES

Measuring of volume of shampoo was made by weighing the products and determining its density with metal pycnometer. For calculating mass of the products the average tare was used.

Nominal quantity of prepackages (shampoo): 1000 ml
Calibrated pycnometer: $V_{pic}$ = 100 ml (100,027 ml ± 0,031 ml) $\Rightarrow$ $U_{(k=2)}$ = 0,031 ml
$\Rightarrow u(V_{pic})$ = 0,0155 ml

Measurements:
Mass of products - net mass ($m_N$): 1024,96 g
Average tare $(\overline{m_T})$: 60,80 g $\Rightarrow$ mass of prepackages – gross mass $(m_G)$: 1085,76 g
Standard deviation of tare $(s_T)$: 0,86 g
Number of samples for tare determination ($n_T$): 10
Number of samples for average gross mass determination ($n_G$): 50

Mass of sample in the pycnometer ($m_d$): 101,47 g
Average density of the sample (ρ): 1,015 g/ml
Number of samples for density determination ($n_d$): 3
Standard deviation of density determination ($s_d$): 8,46 $\cdot 10^{-5}$ g/ml

### 3.1. Verified NAWI

*e* = 0,1 g; *d* = 0,01 g; class II
min = 0,5 g; max = 5100 g

3.1.1 Determination of tare:

$$u(\overline{m_T})^2 = \left(\frac{mpes_T}{\sqrt{3}}\right)^2 + 2\left(\frac{d_T}{2\sqrt{3}}\right)^2 + \left(\frac{s_T}{\sqrt{n_T}}\right)^2$$

$$mpes_T: \quad \frac{\overline{m_T}}{e} = \frac{60{,}80\ g}{0{,}1\ g} = 608{,}0 \Rightarrow mpe_T = \pm 0{,}5e = 0{,}05\ g \Rightarrow mpes_T = 0{,}1\ g$$ [h]

$$u(\overline{m_T})^2 = \left(\frac{0{,}1\ g}{\sqrt{3}}\right)^2 + 2\left(\frac{0{,}01\ g}{2\sqrt{3}}\right)^2 + \left(\frac{0{,}86\ g}{\sqrt{10}}\right)^2$$

$$u(\overline{m_T}) = 0{,}278047\ g$$

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[h] The maximum permissible errors in service shall be twice of the maximum permissible errors on initial verification: 2mpe = mpes

3.1.2 Determination of mass of a single prepackage (gross mass):

$$u(m_G)^2 = \left(\frac{mpes_G}{\sqrt{3}}\right)^2 + 2\left(\frac{d_G}{2\sqrt{3}}\right)^2$$

$$mpes_G: \quad \frac{m_G}{e} = \frac{1085{,}76\ g}{0{,}1\ g} = 10857{,}6 \Rightarrow mpes_G = \pm 1e = 0{,}1\ g \Rightarrow mpes_G = 0{,}2\ g$$

$$u(m_G)^2 = \left(\frac{0{,}2\ g}{\sqrt{3}}\right)^2 + 2\left(\frac{0{,}01\ g}{2\sqrt{3}}\right)^2$$

$$u(m_G) = 0{,}115542\ g$$

3.1.3 Calculation of mass of product in a single prepackage (net mass)

$$u(m_N)^2 = u(m_G)^2 + u\left(\overline{m_T}\right)^2$$

$$u(m_N)^2 = (0{,}115542\ g)^2 + (0{,}278047\ g)^2$$

$$u(m_N) = 0{,}301098\ g$$

3.1.4 Determination of density of products

$$u(\rho)^2 = (u(m_d)\cdot c(m_d))^2 + (u(V_{pic})\cdot c(V_{pic}))^2 + \left(\frac{s_d}{\sqrt{n_d}}\right)^2$$

$$u(m_d)^2 = \left(\frac{mpes_d}{\sqrt{3}}\right)^2 + 2\left(\frac{d_d}{2\sqrt{3}}\right)^2$$

$$mpes_d: \quad \frac{m_d}{e} = \frac{101{,}47\ g}{0{,}1\ g} = 1014{,}7\ g \Rightarrow mpes_d = \pm 0{,}5e = 0{,}05\ g \Rightarrow mpes_d = 0{,}1\ g$$

$$u(m_d)^2 = \left(\frac{0{,}1\ g}{\sqrt{3}}\right)^2 + 2\left(\frac{0{,}01\ g}{2\sqrt{3}}\right)^2$$

$$u(m_d) = 0{,}057879\ g$$

$$c(m_d) = \frac{\partial\rho}{\partial m} = 0{,}99985\cdot\frac{1}{V_{pic}} = 0{,}99985\cdot\frac{1}{100{,}027\ ml} = 0{,}009996\ ml^{-1}$$

$$c(V_{pic}) = \frac{\partial\rho}{\partial V} = -0{,}99985 \cdot \frac{m_d}{V_{pic}{}^2} = -0{,}99985 \cdot \frac{101{,}47\ g}{100{,}027^2\ ml^2} = -0{,}01014\ gml^{-2}$$

$$u(\rho)^2 = (0{,}057879\ g \cdot 0{,}009996\ ml^{-1})^2 + (0{,}0155\ ml \cdot (0{,}01014\ gml^{-2}))^2 + \left(\frac{8{,}46 \cdot 10^{-5}}{\sqrt{3} \cdot ml}\right)^2$$

$$u(\rho) = 0{,}000602\ gml^{-1}$$

### 3.1.5 Combined uncertainty, $u_c(V)$

$$u_c(V) = \frac{m_N}{\rho}\sqrt{\left(\frac{u(m_N)}{m_N}\right)^2 + \left(\frac{u(\rho)}{\rho}\right)^2}$$

$$u_c(V) = \frac{1024{,}96\ g}{1{,}015\ gml^{-1}} \cdot \sqrt{\left(\frac{0{,}301098\ g}{1024{,}96\ g}\right)^2 + \left(\frac{0{,}000602\ gml^{-1}}{1{,}015\ gml^{-1}}\right)^2}$$

$$u_c(V) = 0{,}667921\ ml$$

### 3.1.6 Expanded uncertainty, U

$$U = k \cdot u_c(V)$$

$$U_{(k=2)} = 2 \cdot u_c(V)$$

$$U_{(k=2)} = 2 \cdot 0{,}667921\ ml$$

$U_{(k=2)}$ = 1,335842 = 1,34[i]

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[i] When calculating the expanded uncertainty common practice the figure is rounded up; rules and more information on is available in EA4/02.

### 3.2. Calibrated NAWI

d = 0,01 g
max = 5100 g
UM (from certificate of calibration of NAWI), $U_{k=2}$ = 0,0047 g + 3,90 · $10^{-5}$ · *m* (*m* is a mass of load)

⇒ u(WI) = (0,0047 g + 3,90 · $10^{-5}$ · *m*)/2

3.2.1 Determination of tare

$$u(\overline{m_T})^2 = u(WI_T)^2 + \left(\frac{s_T}{\sqrt{n_T}}\right)^2$$

$$u(WI_T) = (0{,}0047\ g + 3{,}90 \cdot 10^{-5} \cdot 60{,}80\ g)/2$$

$$u(WI_T) = 0{,}003536\ g$$

$$u(\overline{m_T})^2 = (0{,}003536\ g)^2 + \left(\frac{0{,}86\ g}{\sqrt{10}}\right)^2 = 0{,}073973\ g^2$$

$$u(\overline{m_T}) = 0{,}271979\ g$$

3.2.2 Determination of mass of prepackage (gross mass)

$$u(m_G)^2 = u(WI_G)^2$$

$$u(WI_G) = (0{,}0047 g + 3{,}90 \cdot 10^{-5} \cdot 1085{,}76 g)/2 = 0{,}023522 g$$

$$u(m_G) = 0{,}023522\ g$$

3.2.3 Calculation of mass of product in the prepackage (net mass)

$$u(m_N)^2 = u(m_G)^2 + u(m_T)^2$$

$$u(m_N)^2 = (0{,}023522\ g)^2 + (0{,}271979\ g)^2$$

$$u(m_N) = 0{,}272994\ g$$

3.2.4 Determination of density

$$u(\rho)^2 = (u(m_d) \cdot c(m_d))^2 + (u(V_{pic}) \cdot c(V_{pic}))^2 + \left(\frac{s_d}{\sqrt{n_d}}\right)^2$$

$$u(m_d) = u(WI_d)$$

$$u(WI_d) = (0{,}0047\ g + 3{,}90 \cdot 10^{-5} \cdot 101{,}47\ g)/2 = 0{,}004329\ g$$

$$c(m_d) = \frac{\partial \rho}{\partial m} = 0{,}99985 \cdot \frac{1}{V_{pic}} = 0{,}99985 \cdot \frac{1}{100{,}027\ ml} = 0{,}009996\ ml^{-1}$$

$$c(V_{pic}) = \frac{\partial \rho}{\partial V} = -0{,}99985 \cdot \frac{m_d}{V_{pic}{}^2} = -0{,}99985 \cdot \frac{101{,}47 g}{100{,}027^2\ ml^2} = -0{,}01014\ gml^{-2}$$

$$u(\rho)^2 = (0{,}004329\ g \cdot 0{,}009996\ ml^{-1})^2 + (0{,}0155\ ml \cdot (-0{,}01014\ gml^{-2}))^2 + \left(\frac{8{,}46 \cdot 10^{-5}}{\sqrt{3} \cdot ml}\right)^2$$

$$u(\rho) = 0{,}0001702\ gml^{-1}$$

3.2.5 Combined uncertainty, $u_c(V)$

$$u_c(V) = \frac{m_N}{\rho} \sqrt{\left(\frac{u(m_N)}{m_N}\right)^2 + \left(\frac{u(\rho)}{\rho}\right)^2}$$

$$u_c(V) = \frac{1024{,}96\ g}{1{,}015\ gml^{-1}} \cdot \sqrt{\left(\frac{0{,}272994\ g}{1024{,}96\ g}\right)^2 + \left(\frac{0{,}0001702\ gml^{-1}}{1{,}015 gml^{-1}}\right)^2}$$

$$u_c(V) = 0{,}317812\ ml$$

3.2.5.1 Effective degrees of freedom, $\nu_{eff}$

In order to obtain a value for *k* it is necessary to obtain an estimate of the effective degrees of freedom, $\nu_{eff}$ of the combined standard uncertainty $u_c(y)$. The GUM recommends that the Welch-Satterthwaite equation is used to calculate a value for $\nu_{eff}$ based on the degrees of freedom, $\nu_i$ , of the individual standard uncertainties, $u_i(y)$; therefore

$$\nu_{eff} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y)}{\nu_i}}$$

In this instance where $u_i(y)$ is $u_c(V)$, and substituting the data given:

$$\nu_{eff} = \frac{(0{,}317821\,ml)^4}{\dfrac{\left(\dfrac{0{,}271956\ g}{1{,}015\ gml^{-1}}\right)^4}{9} + \dfrac{\left(\dfrac{0{,}0001702\ g}{1{,}015\ gml^{-1}} + \dfrac{0{,}0001702\ g}{1{,}015\ gml^{-1} + 0{,}0000488\ gml^{-1}}\right)^4}{2}}$$

$$\nu_{eff} = 17{,}8$$

From the t-distribution table $k_{95\%}$ = 2,15

3.2.6 Expanded uncertainty, U

$$U = k \cdot u_c(V)$$

$$U_{(k=95\%)} = 2{,}15 \cdot u_c(V)$$

$$U_{(k=95\%)} = 2 \cdot 0{,}317812\ ml$$

$U_{(k=95\%)}$ = 0,683296 ml = 0,69 ml

## 4. CONCLUSIONS

As already said, the uncertainty of measurement (interpreted as the expanded uncertainty) made in measuring the actual quantity of product in prepackages shall not exceed one-fifth of the tolerable negative error for the nominal quantity of the prepackage. In the example in chapter 3 the nominal quantity was 1000 ml. TNE for this quantity is to 15 ml therefore one-fifth is 3 ml. In both cases calculated expanded uncertainty was under that value.

The single difference between both examples was the balance – in first case verified and in the second case calibrated NAWI – and because of the difference in uncertainty of measurement due to the balance there is a need to take into account the effective degrees of freedom to ensure that an appropriate *k* factor is used for the 95% confidence level.